Problem Paper II:3

Question 1

The velocities of individual galaxies don't obey Hubble's Law exactly – galaxies have 'peculiar velocities' (largely resulting from gravitational interactions with neighbouring galaxies) in addition to 'cosmological velocities' resulting from the Hubble flow.
Suppose that a typical peculiar velocity is 500 km s^{-1}. How far away would a galaxy have to be if you wanted to use it to determine the Hubble constant with an uncertainty of 10 per cent? (Assume $H_0 = 72 \text{ km s}^{-1}\text{Mpc}^{-1}$, and that the distance and velocity of the galaxy can be measured perfectly accurately; express your answer in Mpc.)

For the given value of H_0, calculate a value for the Hubble time. (Express your answer in years, to 2 significant figures.)

Question 2

The velocity of recession of a galaxy is very often expressed in terms of its redshift, z. If $v \ll c$ we can convert from redshift to velocity by using the usual Doppler formula,

$$1 + z \equiv \frac{\lambda}{\lambda_o} \simeq 1 + \frac{v}{c} \quad \text{(i.e., } v \simeq cz\text{);}$$

otherwise the relativistic form must be used:

$$1 + z \equiv \frac{\lambda}{\lambda_o} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}}.$$

To show that v and cz are significantly different even for quite small redshifts, rearrange the special-relativistic formula for z into an equation for v/c. Plot a graph of v/c vs. z for z in the range 0 to 3; for comparison, also plot the line for the (unrealistic, hypothetical, non-relativistic) case $v = cz$. Put redshift on the x axis.
[*Hint:* to obtain an expression for v/c, try writing $(1 - v^2/c^2)$ as $(1 - v/c)(1 + v/c)$.]